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DKP Oscillator with Spin-0 in Three-dimensional Noncommutative Phase Space

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Abstract The DKP equation with Dirac oscillator potential for spin-0 particles has been studied when both space-space noncommutativity and momentum-momentum noncommutativity are considered. The exact wave functions and corresponding energy levels have been found. Due to the noncommutative effect, the energy spectrum is not degenerate.

Keywords DKP equation · Noncommutative phase space · Exact solutions

1 Introduction

The DKP equation that describes the interaction of $S=0$ and $S=1$ hadrons with difference nuclei is not new and dates back to the 1930s [12, 26]. Historically, considering greater algebraic complexity of DKP formulation and the equivalence of DKP approach to the KG and Proca descriptions in on-shell situations, there are few papers that are interested in the investigation of the DKP equation. However, in the past decade, this supposed equivalence began to be studied in several situations involving breaking of symmetries and hadronic process that shows that in some cases the DKP equation and KG equation can give different results. There has been a growing interest in studying the DKP equation [5–8, 18, 24, 28]. On other hand, recently the issue of noncommutative quantum mechanics has been extensively discussed [9, 33–35]. This was initially motivated by studies of the low energy effective theory of D-brane with a nonzero Neveu-Schwarz B field background. Many efforts have been devoted to the various aspects of noncommutative quantum mechanics, such as Quantum Hall

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effect [10, 14], Landau problem on noncommutative plane [1, 11, 16], the two-dimensional quantum system with arbitrary central potential [2, 17], and the DKP oscillator in a noncommutative space [15], etc. The noncommutative phase space is characterized by the fact that their coordinate operators satisfy the equation [4],

$$[\hat{x}^{\mu}, \hat{x}^{\nu}] = i\theta^{\mu\nu}, \tag{1}$$

where $\theta^{\mu\nu}$ is a n antisymmetric tensor and plays an analogous role to $\hbar$ in the usual quantum mechanics. also, non-commutativity gauge theory and non-commutativity field theory are widely studied based on Weyl-Moyal correspondence, in which all products are replaced by the star product in order to obtain their non-commutativity action [23]

$$\begin{aligned}(f * g)(x, p) &= \exp\left(\frac{i}{2}\theta^{ij}\partial_i^x\partial_j^x + \frac{i}{2}\eta^{ij}\partial_i^p\partial_j^p\right) f(x, p)g(x, p) \\ &= f(x, p)g(x, p) + \frac{i}{2}\theta^{ij}\partial_i^x f(x, p)\partial_j^x g(x, p)\Big|_{x_i = x_j} \\ &\quad + \frac{i}{2}\eta^{ij}\partial_i^p f(x, p)\partial_j^p g(x, p)\Big|_{p_i = p_j},\end{aligned} \tag{2}$$

where the constant parameter θ^{ij} and η^{ij} is an anti-symmetric tensor and which represents the noncommutativity of the space, f and g are the infinitely differentiable functions.

In other words, In the noncommutative phase space, (2) of the noncommutative algebra [21] can be written as:

$$\begin{aligned}&[\hat{x},^i \hat{x}^j] = i\theta^{ij}, \qquad [\hat{p}^i, \hat{p}^j] = i\eta^{ij}, \qquad [\hat{x}^i, \hat{p}^j] = i\hbar\delta^{ij}, \qquad \hbar_{eff} = \hbar\left(1 + \frac{\theta\eta}{4\hbar^2}\right) \\ &(i, j = 1, 2, 3),\end{aligned} \tag{3}$$

where θ^{ij} and η^{ij} are noncommutativity parameters. One possible way of implementing algebra equation (3) is to construct the noncommutative variables $\{\hat{x}_1, \hat{x}_2, \hat{x}_3, \hat{p}_1, \hat{p}_2, \hat{p}_3\}$ from the commutative $\{x_1, x_2, x_3, p_1, p_2, p_3\}$ by the linear transformation:

$$\hat{x}_i = x_i - \frac{1}{2\hbar}\theta^{ij}p_j, \qquad \hat{p}_i = p_i + \frac{1}{2\hbar}\eta^{ij}x_j \quad (i, j = 1, 2, 3). \tag{4}$$

The parameters of noncommutativity can be identified with two vectors $\theta^{ij} = \varepsilon^{ijk}\theta_k$ and $\eta^{ij} = \varepsilon^{ijk}\eta_k$. Letting $\theta_k = (0, 0, \theta)$ and $\eta_k = (0, 0, \eta)$.

As we know, in spite of the great number of papers published regarding the DKP equation, many works was restricted to commutative or non commutative space and none has reported its exact solutions in noncommutative phase space. The goal of this study is to find the exact wave functions and the energy levels of DKP equation with Dirac oscillator potential for spin-0 particles [15, 27, 29] in noncommutative phase space.

The plan of this paper is organized as follows. In Sect. 2, we obtain the exact solutions of the DKP equation in noncommutative phase space. Section 3 will be devoted to the conclusion.

2 Exact Solutions of the DKP Equation with Dirac Oscillator Potential for Spin-0 in Noncommutative Phase Space

Generally, the first order relativistic DKP equation [13, 25, 31, 32] for a free spin-1 or spin-0 particle of mass m is

$$(i\beta^{\mu}\partial_{\mu} - m)\psi = 0, \tag{5}$$

where β^{μ} ($\mu = 0, 1, 2, 3$) matrices satisfy the commutation relation

$$\beta^{\mu}\beta^{\nu}\beta^{\lambda} + \beta^{\lambda}\beta^{\nu}\beta^{\mu} = g^{\mu\nu}\beta^{\lambda} + g^{\nu\lambda}\beta^{\mu}, \quad g^{\mu\nu} = (1, -1, -1, -1), \tag{6}$$

which defines the so-called Duffin-Kemmer-Petiau algebra. For spin-1 particles, β^{μ} are 10×10 matrices given by [20]

$$\beta^0 = \begin{bmatrix} o_{3\times3} & o_{3\times3} & -I_{3\times3} & \bar{0}^T \\ o_{3\times3} & o_{3\times3} & o_{3\times3} & \bar{0}^T \\ -I_{3\times3} & o_{3\times3} & o_{3\times3} & \bar{0}^T \\ \bar{0} & \bar{0} & \bar{0} & 0 \end{bmatrix}, \qquad \beta^l = \begin{bmatrix} o_{3\times3} & o_{3\times3} & o_{3\times3} & k^{lT} \\ o_{3\times3} & o_{3\times3} & -is^l_{3\times3} & \bar{0}^T \\ o_{3\times3} & is^l_{3\times3} & o_{3\times3} & \bar{0}^T \\ k^l & \bar{0} & \bar{0} & 0 \end{bmatrix}, \tag{7}$$

where $l = 1, 2, 3$ and $s^l_{3\times3}$ are the usual 3×3 spin-1 matrices which can be give as follows

$$s^1_{3\times3} = i\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}, \quad s^2_{3\times3} = i\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}, \quad s^3_{3\times3} = i\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{8}$$

and the other matrices in (7) are given as follows

$$o_{3\times3} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad I_{3\times3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \begin{cases} k^1 = (100) \\ k^2 = (010) \\ k^3 = (001) \end{cases}, \quad \bar{0} = (000). \tag{9}$$

For spin-0 particles, β^{μ} are 5×5 matrices given by

$$\beta^0 = \begin{bmatrix} \theta & \bar{0} \\ \bar{0}^T & 0 \end{bmatrix}, \qquad \beta^i = \begin{bmatrix} \tilde{0} & \rho^i \\ -\rho^{iT} & 0 \end{bmatrix}, \tag{10}$$

with $\tilde{0}, \bar{0}, 0$ as 2×2, 2×3, 3×3 zero matrices, respectively, and the other matrices in (10) are given as follows

$$\theta = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad -\rho^1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad -\rho^2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad -\rho^3 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}. \tag{11}$$

In this paper, we mainly consider the DKP equation with Dirac oscillator potential [3] for spin-0 in noncommutative phase space. In this case, operator $\hat{p}$ in the free DKP equation could be substituted by $\hat{p} - im\omega\eta^0\hat{r}$, with $\eta^0 = 2(\beta^0)^2 - I$ and $(\eta^0)^2 = I_{5\times5}$ being the 5×5 unit matrix. So the DKP equation with Dirac oscillator interaction in three-dimensional noncommutative phase space can be written as follows:

$$[\beta^0 E - c\beta^1(\hat{p}_1 - im\omega\eta^0\hat{x}_1) - c\beta^2(\hat{p}_2 - im\omega\eta^0\hat{x}_2) - c\beta^3(\hat{p}_3 - im\omega\eta^0\hat{x}_3) - mc^2]\psi = 0, \tag{12}$$

where the state ψ is a five-component wave function of the DKP equation, which can be given by

$$\psi(x_1, x_2, x_3) = (\psi_1\psi_2\psi_3\psi_4\psi_5)^T. \tag{13}$$

Substituting (13) into (12), we can obtain the following coupled equations:

$$-mc^2\psi_1 + E\psi_2 + c(\hat{p}_1 + im\omega\hat{x}_1)\psi_3 + c(\hat{p}_2 + im\omega\hat{x}_2)\psi_4 + c(\hat{p}_3 + im\omega\hat{x}_3)\psi_5 = 0, \tag{14}$$

$$\begin{aligned} &mc^2\psi_2 - E\psi_1 = 0, && mc^2\psi_4 + c(\hat{p}_2 - im\omega\hat{x}_2)\psi_1 = 0,\\ &mc^2\psi_3 + c(\hat{p}_1 - im\omega\hat{x}_1)\psi_1 = 0, && mc^2\psi_5 + c(\hat{p}_3 - im\omega\hat{x}_3)\psi_1 = 0. \end{aligned}$$

The (14) is equivalent to the following equations

$$\begin{aligned} &\psi_2 = \frac{E}{mc^2}\psi_1, \quad \psi_3 = -\frac{c(\hat{p}_1 - im\omega\hat{x}_1)}{mc^2}\psi_1,\\ &\psi_4 = -\frac{c(\hat{p}_2 - im\omega\hat{x}_2)}{mc^2}\psi_1, \quad \psi_5 = -\frac{c(\hat{p}_3 - im\omega\hat{x}_3)}{mc^2}\psi_1,\\ &[c^2(\hat{p}_1 + im\omega\hat{x}_1)(\hat{p}_1 - im\omega\hat{x}_1) + c^2(\hat{p}_2 + im\omega\hat{x}_2)(\hat{p}_2 - im\omega\hat{x}_2)]\psi_1\\ &\quad + [c^2(\hat{p}_3 + im\omega\hat{x}_3)(\hat{p}_3 - im\omega\hat{x}_3) + m^2c^4 - E^2]\psi_1 = 0. \end{aligned} \tag{15}$$

Making use of the (4), the last equation in the (15) can be written as

$$\begin{aligned} &\left[\left(\frac{1}{2m} + \frac{m\omega^2\theta^2}{8\hbar^2}\right)(p_1^2 + p_2^2) + \left(\frac{m\omega^2}{2} + \frac{\eta^2}{8m\hbar^2}\right)(x_1^2 + x_2^2) + \frac{p_3^2}{2m} + \frac{m\omega^2 x_3^2}{2}\right]\psi_1\\ &\quad - \left(\frac{\eta + m^2\omega^2\theta}{2m\hbar}L_z + \frac{E^2 - m^2c^4 + 2mc^2\hbar_{eff}\omega + mc^2\hbar\omega}{2mc^2}\right)\psi_1 = 0. \end{aligned} \tag{16}$$

By taking the following wave function

$$\psi_1(\rho, \phi, x_3) = \chi(\rho)e^{i|m_l|\phi}\psi^{osc}(x_3), \quad m_l = 0, \pm 1, \pm 2, \pm 3, \ldots, \tag{17}$$

and substituting (17) into (16),we can easily arrive at

$$\left[\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_3^2} + \left(E_{n_{x_3}} - \frac{1}{2}m\omega^2 x_3^2\right)\right]\psi^{osc}(x_3) = 0, \tag{18}$$

$$\left[\frac{\hbar^2}{2\bar{m}}\left(\frac{d^2}{d\rho^2} + \frac{1}{\rho}\frac{d}{d\rho} - \frac{m_l^2}{\rho^2}\right) + \left(\bar{E} - E_{n_{x_3}} - \frac{1}{2}\bar{m}\bar{\omega}^2\rho^2\right)\right]\chi(\rho) = 0, \tag{19}$$

where

$$\begin{aligned} &\bar{m} = \frac{4m\hbar^2}{4\hbar^2 + m^2\omega^2\theta^2},\\ &\bar{\omega} = \frac{1}{4m\hbar^2}\sqrt{(4m^2\hbar^2\omega^2 + \eta^2)(4\hbar^2 + m^2\theta^2\omega^2)},\\ &\bar{E} = \frac{E^2 - m^2c^4 + 2mc^2\hbar_{eff}\omega + mc^2\hbar\omega + \eta c^2 m_l + m^2c^2\omega^2\theta m_l}{2mc^2}. \end{aligned}$$

The (18) is well-known the one-dimensional Schrödinger equation of oscillator and we can easily get

$$\psi^{osc}(x_3) = \sqrt{\frac{\sqrt{m\omega}}{2^{n_{x_3}} \cdot \sqrt{\pi\hbar} \cdot n_{x_3}!}} e^{-\frac{m\omega}{2\hbar}x_3^2} \mathrm{H}_{n_{x_3}}\left(\sqrt{\frac{m\omega}{\hbar}} x_3\right), \tag{20}$$

$$E_{n_{x_3}} = \hbar\omega\left(n_{x_3} + \frac{1}{2}\right), \quad n_{x_3} = 0, 1, 2, 3, \ldots. \tag{21}$$

In the (20),

$$\mathrm{H}_{n_{x_3'}}\left(\sqrt{\frac{m\omega}{\hbar}} x_3\right)$$

is Hermit polynomial.

In order to solve (19),make the change of $y = m\omega\rho^2/2\hbar$ and (19) can be expressed as

$$\left[y\frac{d^2}{dy^2} + \frac{d}{dy} + \left(\beta - y - \frac{m_l^2}{4y}\right)\right]\chi(y) = 0, \tag{22}$$

where $\beta = (\tilde{E} - E_{n_{x_3}})/\hbar\omega$. Introducing a new variable $\gamma = |m_l| + 1$ and $\alpha = -(\beta - \gamma)/2$. and considering the following radial wave function

$$\chi(\xi) = e^{-\xi}\xi^{|m_l|/2} f(\xi). \tag{23}$$

Equation (22) can be written as

$$\xi\frac{d^2 f(\xi)}{d\xi^2} + [\gamma - 2\xi]\frac{df(\xi)}{d\xi} - 2\alpha f(\xi) = 0. \tag{24}$$

Taking $z = 2\xi$, and inserting it into (24), we have following equation

$$z\frac{d^2 f(z)}{dz^2} + (\gamma - z)\frac{df(z)}{dz} - \alpha f(z) = 0. \tag{25}$$

(25) is nothing but the confluent hypergeometrics equation [19], and considering the boundary condition that $z \to 0(\rho \to 0)$ leads $\chi(z)$ tending to finite, Its solutions are well- known confluent hypergeometric function type

$$f(z) = N\mathrm{F}(\alpha, \gamma, z). \tag{26}$$

Considering the boundary condition [21] $z \to \infty(\rho \to \infty)$, $\chi(z) \to 0$ we obtain

$$\alpha = -[(\bar{E} - E_{n_{x_3}})/\hbar\omega - |m_l| - 1]/2 = -n_\rho, \quad n_\rho = 0, 1, 2, 3, \ldots, \tag{27}$$

and the function ψ_1 can be written in following form

$$\psi_1(\rho, \phi, x_3) = N\rho^{|m_l|} e^{i|m_l|\phi} \exp\left(-\frac{\bar{m}\bar{\omega}}{2\hbar}\rho^2 - \frac{m\omega}{2\hbar}x_3^2\right)$$
$$\times \mathrm{F}\left(-n_\rho, |m_l| + 1, \frac{\bar{m}\bar{\omega}}{\hbar}\rho^2\right)\mathrm{H}_{n_{x_3}}\left(\sqrt{\frac{m\omega}{\hbar}} x_3\right), \tag{28}$$

where N is the normalization constant.

Noting that $\bar{E}=[E^2-m^2c^4+2mc^2\hbar_{eff}\omega+mc^2\hbar\omega+\eta c^2m_l+m^2c^2\omega^2\theta m_l]/2mc^2$, $\alpha=-(\beta-\gamma)/2$ and $\alpha=-[(\bar{E}-E_{n_{x_3}})/\hbar\omega-|m_l|-1]/2=-n_\rho$, the eigenvalue of DKP equation with Dirac oscillator potential can be obtained in form

$$\begin{aligned}E^2_{n_\rho m_l x_3}&=2mc^2\hbar\bar{\omega}(2n_\rho+|m_l|+1)+2mc^2\hbar\omega\left(n_{x_3}+\frac{1}{2}\right)+m^2c^4\\&\quad-c^2(\eta+m^2\omega^2\theta)m_l-2mc^2\hbar_{eff}\omega-mc^2\hbar\omega.\end{aligned}\tag{29}$$

From (29), we can see that the energy spectrum of massive spin-0 particles is exact and not degenerate due to the noncommutative effect. Making use of the solution of component $\psi_1(\rho,\phi,x_3)$ and considering (15), finally the corresponding total wave function $\psi(\rho,\phi,x_3)$ can be deduced. As a consequence, the total wave function $\psi(\rho,\phi,x_3)$ can be expressed

$$\psi(\rho,\phi,x_3)=\Lambda\begin{bmatrix}\mathrm{H}\\ E\mathrm{H}/mc^2\\ i(\beta e^{i\phi}-\gamma e^{-i\phi})\mathrm{H}\\ (\beta e^{i\phi}+\gamma e^{-i\phi})\mathrm{H}\\ i2n_{x_3}\sqrt{\hbar\omega/mc^2}\mathrm{H}'\end{bmatrix}\mathrm{F}_1+\frac{i\bar{m}\bar{\omega}\rho n_\rho\Lambda}{mc(|m_l|+1)}\begin{bmatrix}0\\0\\-[\vartheta_+e^{i\phi}+\vartheta_-e^{-i\phi}]\mathrm{H}\\ i[\vartheta_+e^{i\phi}-\vartheta_-e^{-i\phi}]\mathrm{H}\\0\end{bmatrix}\mathrm{F}_2,\tag{30}$$

$$\Lambda=N\rho^{|m_l|}e^{-\frac{\bar{m}\bar{\omega}\rho^2}{2\hbar}-\frac{m\omega x_3^2}{2\hbar}+i|m_l|\phi},\quad \beta=\frac{-1}{2mc}\left[\frac{\bar{m}m\bar{\omega}\omega\theta-\eta}{2\hbar}+(\bar{m}\bar{\omega}-m\omega)\right]\rho,$$

$$\gamma=\frac{-1}{2mc}\left[\frac{\bar{m}m\bar{\omega}\omega\theta-\eta}{2\hbar}-(\bar{m}\bar{\omega}-m\omega)\right]\rho+\frac{-\hbar}{mc}\left(1-\frac{m\omega\theta}{2\hbar}\right)\frac{|m_l|}{\rho},$$

$$\mathrm{F}_1=\mathrm{F}\left(-n_\rho,1+|m_l|,\frac{\bar{m}\bar{\omega}\rho^2}{\hbar}\right),\quad \mathrm{F}_2=\mathrm{F}\left(1-n_\rho,2+|m_l|,\frac{\bar{m}\bar{\omega}\rho^2}{\hbar}\right),$$

$$\vartheta_+=1+\frac{m\omega\theta}{2\hbar},\quad \vartheta_-=1-\frac{m\omega\theta}{2\hbar},\quad \bar{\omega}=\frac{\sqrt{(4m^2\hbar^2\omega^2+\eta^2)(4\hbar^2+m^2\theta^2\omega^2)}}{4m\hbar^2},$$

$$\mathrm{H}=\mathrm{H}_{n_{x_3}}\left(\sqrt{\frac{m\omega}{\hbar}}x_3\right),\quad \mathrm{H}'=\mathrm{H}_{n_{x_3}-1}\left(\sqrt{\frac{m\omega}{\hbar}}x_3\right),\quad \bar{m}=\frac{4m\hbar^2}{4\hbar^2+m^2\omega^2\theta^2}.$$

This total wave function must satisfy the normalization condition [30]

$$\int\bar{\psi}(\rho,\phi)\beta^0\psi(\rho,\phi)\rho d\rho d\phi=1.\tag{31}$$

Substituting (30) into (31) and after some computation, we can obtain the normalization constant N as

$$N=\sqrt{\frac{\frac{E}{mc^2\sqrt{\pi^3}\cdot 2^{n_{x_3}+1}\cdot n_{x_3}!}\left(\frac{\bar{m}\bar{\omega}}{\hbar}\right)^{|m_l|+1}\left(\frac{m\omega}{\hbar}\right)^{1/2}}{\sum_{n=0}^{n_\rho}\sum_{k=0}^{n}\frac{(n+|m_l|)!(-n_\rho)_k(-n_\rho)_{n-k}}{k!(n-k)!(1+|m_l|)_k(1+|m_l|)_{n-k}}}},\quad \begin{cases}n_\rho=0,1,2,3,\ldots\\ n_{x_3}=0,1,2,3,\ldots\\ m_l=0,\pm1,\pm2,\ldots\end{cases}.\tag{32}$$

3 Discussion

The DKP equation with Dirac oscillator potential for spin-0 particles has been studied in the noncommutative phase space. Its wave function and corresponding energy level can be obtained, its normalization constant can be obtained also. From the results of this paper, if we let the parameter η is equal to zero, the result of this paper will be reduced to the result as [15] in noncommutative space, if we just consider two-dimentional noncommutative phase space, the result of this paper will be reduced to the result as [22], and will be reduced to the result in commutative space for $\eta = \theta = 0$.

4 Conclusion

The DKP equation with Dirac oscillator potential for spin-0 particles has been studied when both space-space noncommutativity and momentum-momentum noncommutativity are considered. The exact wave functions and corresponding energy levels are obtained. Due to the noncommutative effect, the energy spectrum of massive spin-0 particles is not degenerate. The result obtained here can be reduced to the result in commutative space for $\eta = 0$ and $\theta = 0$.

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